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Reverse Hölder-type inequalities: Applications to the Slicing problem

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SUMMARY

Given a convex body $K \subset \mathbb{R}^n$, i.e., a compact convex set with non empty interior, and an integrable function $f : K \to [0, \infty)$, Hölder's inequality states that for any $0 < p < q$,

$$\left(\frac{1}{|K|} \int_K f(x)^p \right)^{1/p} \leq \left(\frac{1}{|K|} \int_K f(x)^q \right)^{1/q}.$$

Berwald proved in [2] that under concavity asumption of f, a reverse inequality can be obtained, showing that for any $0 < p < q$,

$$\left(\frac{\binom{n+p}{n}}{|K|} \int_K f(x)^p \right)^{1/p} \geq \left(\frac{\binom{n+q}{n}}{|K|} \int_K f(x)^q \right)^{1/q}.$$

By analyzing Berwald's proof, we obtain an inequality that improves the inclusion relation among convex bodies in a family defined by K. Ball in [1] associated to log-concave functions, in case that they verify better concavity conditions.

In the second part, we will present the Slicing problem. This conjecture asks if there exists an absolute constant $c > 0$ such that for any dimension $n \in \mathbb{N}$ and any centered convex body $K \subset \mathbb{R}^n$ of volume 1, one can find a hyperplane H passing through the origin, such that the $n-1$-dimensional volume of $K \cap H$ is greater than c. We will show an application of the results obtained about Ball bodies in a reduction of the Slicing problem to the centrally symmetric case.

References

[1] K. Ball, *Logarithmically concave functions and sections of convex sets in* $\mathbb{R}^n$, Stud. Math. **88** (1), 69-84 (1988).

[2] L. Berwald, Verallgemeinerung eines Mittelwetzsatzes von J. Favard, *Für positive konkave Funktionen*. Acta Math. 79, 17-37 (1947).

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